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MGP Versus Kochen–Specker Condition in Hidden Variables Theories

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Hidden variables theories for quantum mechanics are usually assumed to satisfy the *KS condition*. The Bell–Kochen–Specker theorem then shows that these theories are necessarily contextual. But the KS condition can be criticized from an operational viewpoint, which suggests that a weaker condition (MGP) should be adopted in place of it. This leads one to introduce a class of *hidden parameters theories* in which contextuality can, in principle, be avoided, since the proofs of the Bell–Kochen–Specker theorem break down. A simple model recently provided by the author for an objective interpretation of quantum mechanics can be looked at as a noncontextual hidden parameters theory, which shows that such theories actually exist.

KEY WORDS: quantum mechanics; KS condition; hidden variables; hidden parameters.

1. INTRODUCTION

Kochen and Specker (1967) have shown that, for every statistical theory, a phase space Ω of hidden states and a probability measure μ on Ω can be introduced such that a state uniquely determines the values of all observables and the statistical predictions of the theory are reproduced. In this broad sense, therefore, *hidden parameters theories* exist for any statistical theory, hence for quantum mechanics (QM).

According to the standard interpretation, however, QM also yields predictions for properties of individual samples of general physical systems (briefly, *individual systems*, or *physical objects*). For instance, one says that the values of mutually compatible observables can be measured simultaneously on an individual system. Yet, it is well known that a number of difficulties occur when trying to interpret the statistical predictions in terms of individual systems, so that some scholars foster a statistical interpretation of QM only. But if one accepts that also individual systems enter into play in the interpretation of QM, some further conditions have

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to be imposed on hidden parameters theories aiming to reproduce all the results of QM (briefly, HPTs), besides the condition that the measure μ must allow one to recover all quantum probabilities. At first sight, one expects that these conditions follow directly from the standard interpretation of QM, but a deeper analysis shows that there is a degree of arbitrariness in choosing them: for, the interpretation of a physical theory (hence, in particular, of QM) is never complete, and new physical conditions may actually establish new, or partially new, interpretative assumptions.

Bearing in mind the earlier remark, the condition that HPTs are usually assumed to fulfill, which constitutes a basic premise for the arguments proving the contextuality and nonlocality of HPTs, is considered in Section 2, and some criticisms that can be raised against it are resumed. Furthermore, it is shown in Section 3 how the earlier criticisms can be avoided by weakening the KS condition: this weakening entails broadening the class of possible HPTs and implies the remarkable result that the theorems mentioned earlier do not hold in the new class, so that *noncontextual* and *local* HPTs may exist in it. Finally, an example of a theory of this kind is provided in Section 4, summarizing the model that has been propounded in some recent papers in order to show that an objective interpretation of QM is possible (Garola, 2002; Garola and Pykacz, 2004).

2. THE KS CONDITION

As anticipated in Section 1, this section focuses on the condition that is introduced by Kochen and Specker as a basic requirement "for the successful introduction of hidden variables." This condition is restated by Mermin (1993) in a very simple form, as follows.

2.1. KS Condition

If a set $(A, B, C, \ldots)$ of mutually commuting observables satisfies a relation of the form $f(A, B, C, \ldots) = 0$ then the values $v(A)$, $v(B)$, $v(C)$, ... assigned to them in an individual system must also be related by $f(v(A), v(B), v(C), \ldots) = 0$.

The existence of some arbitrariness in postulating the KS condition is explicitly recognized by Mermin, who writes, before stating it,

> Here is what I hope you will agree is a plausible set of assumptions for a straightforward hidden variables theory.

Following Kochen and Specker, it is usually assumed that only HPTs satisfying the KS condition can be accepted, so that the name "hidden variables theories" itself understands that this condition is fulfilled. Then, all proofs of the Bell–Kochen–Specker (briefly, Bell-KS) theorem, which states the impossibility of constructing noncontextual hidden variables theories for QM, use the KS condition explicitly and *repeatedly*. Hence, the theorem is proved only for HPTs that satisfy this condition.

Notwithstanding the almost universal acceptance of the KS condition, the fact that it does not follow directly from QM but constitutes an additional interpretative assumption suggests that one should inquire more carefully about its consistence with the rest of the interpretative apparatus of QM. Whenever this inquiry is performed, one sees that this condition seems physically plausible, but there are serious arguments for criticizing its repeated use in the proofs. These arguments have been discussed in a number of papers (see, e.g., Garola, 2000; Garola and Solombrino, 1996a), and cannot be reported here in detail. The core of the criticism, however, can be summarized as follows. The repeated use of the KS condition leads one to consider physical situations in which several relations of the form $f(A, B, C, \ldots) = 0$ are assumed to hold *simultaneously*, though there are observables in some relations that do not commute with other observables appearing in different relations. Hence, one envisages physical situations in which several empirical physical laws[2] (those expressed by the relations themselves) are assumed to be simultaneously valid though they cannot be simultaneously checked. This sounds inconsistent with the operational philosophy of QM.

It is still interesting to observe that also the proofs of nonlocality of QM stand on assuming particular instances of the KS condition. This must however be recognized by direct inspection, since this assumption is not explicit in most cases.

3. A WEAKER CONDITION FOR HPTS

If the criticism to the KS condition is accepted, one can try to replace this condition with a weaker constraint, more respectful of the operational philosophy of QM. To this end, one can start from the basic remark that the hidden variables taken into account by the Bell-KS theorem (in order to disprove their existence) are supposed to determine the values of all observables independently of the environment (*noncontextual hidden variables*). This implies that two kinds of physical situations can be envisaged because of the existence of a compatibility relation on the set of all observables. To be precise, if x is a physical object that is produced in a given state by means of a suitable preparing device, an *accessible physical situation* is envisaged whenever x is assumed to be detected if a measurement is done and possessing some pairwise compatible properties, while a *nonaccessible physical situation* is envisaged whenever x is assumed to

[2] It is well known that every general physical theory, as QM, contains both *theoretical* and *empirical* physical laws. Intuitively, a law is theoretical if it includes theoretical terms or (in the case of QM) noncommuting observables. A law of this kind cannot be checked directly: rather, it must be regarded as a *scheme of laws*, from which empirical laws (that can be directly checked in suitable physical situations) can be deduced.

be not detected if a measurement is done or possessing properties that are not pairwise compatible.[3] Now, note that the physical situations considered at the end of Section 2 are examples of nonaccessible physical situations. The arguments carried out when criticizing the KS condition therefore suggest that this condition should be weakened by restricting its validity to accessible physical situations. Thus, one is led to state the following *Metatheoretical Generalized Principle*.

3.1. MGP

A physical statement expressing an empirical physical law is true in all accessible physical situations, but it may be false (as well as true) in nonaccessible situations.

The earlier principle has been proposed in a number of previous papers. Here, however, the definition of *accessible physical situation* takes into account the possibility that the physical object be not detected, which guarantees consistency (Garola, 2002, 2003; Garola and Pykacz, 2004). Substituting MGP to the stronger KS condition implies considering a class of HPTs that includes the class of hidden variables theories in the standard sense. In this broader class, the proofs of the Bell-KS theorem break down (it can be seen that the same occurs for the proofs of nonlocality, see Garola and Solombrino, 1996b), since the KS condition cannot be applied. Thus, at least in principle, *noncontextual (and local) HPTs are possible*.

It remains to show, however, that such kind of theories actually exist. This existence has been proved in some of the papers mentioned earlier by providing a model for an interpretation of QM that is *objective*, in the sense that any conceivable property of a physical system either is possessed or not by a sample of the system, independently of any measurement. This model (called *SR model*, since it subtends an epistemological attitude that was called *Semantic Realism* in the aforementioned papers) actually does not mention explicitly hidden parameters, but some elements in it can be interpreted as such. Since objectivity implies noncontextuality, these hidden parameters are noncontextual. Moreover, one can show that they do not satisfy the KS condition (which would be prohibited by the Bell-KS theorem), hence they are not hidden variables in the standard sense, but satisfy MGP. Thus, the SR model provides a sample of noncontextual HPT.

Before coming to a brief review of the SR model, note that the fact that it satisfies MGP instead of the KS condition illustrates the price to pay in order to avoid the contextuality of QM: one must admit that empirical physical laws

[3] From an operational viewpoint, an accessible physical situation is characterized by the fact that one can single out a subset (that can be void) of physical objects possessing the desired properties when considering a set of physical objects in a state S; indeed, this can be done by performing a suitable measurement on every physical object in the state S (of course, the state of the objects after the measurement might not coincide with S). No such subset can instead be singled out if a nonaccessible physical situation is envisaged.

may fail to be true whenever one considers physical situations that are classified as nonaccessible because of QM itself. This restriction is theoretically relevant but has no *direct* empirical consequence (it may have some *indirect* empirical consequences, as predicting that the Bell inequalities can be violated also in an objective interpretation of QM, see, e.g., Garola and Pykacz, 2004), and constitutes a (cheap) charge for avoiding old problems and paradoxes in the interpretation of QM. For instance, the *objectification* problem in quantum measurement theory, which remains unsolved also in some sophisticated generalizations of standard QM, as unsharp quantum mechanics (see, e.g., Busch *et al.*, 1991), obviously disappears in an objective interpretation of QM. Analogously, the Schrödinger's cat paradox, the Wigner's friend paradox, etc., also disappear.

4. THE SR MODEL

As anticipated in Section 1, this section is devoted to resume the essential features of the SR model and to illustrate qualitatively how it may happen that some widely accepted results, as the contextuality of QM, may fail to hold in the interpretation of QM provided by the model. This result can be better achieved proceeding by steps, as follows.

(i) States are neatly distinguished from physical properties in the SR model, since they are defined, as in Ludwig (1983), by means of preparation procedures. To be precise, a state is defined as a class of physically equivalent preparation procedures. A physical object in a given state S is then defined by a preparation act, performed by means of a preparation procedure that belongs to the class denoted by S. Furthermore, pure states are represented by vectors of a Hilbert space $\mathcal{H}$ associated with the physical system, as in standard QM.

(ii) Properties are defined as pairs $(\mathcal{A}_0, \Delta)$, where $\mathcal{A}_0$ is a *measurable physical quantity* (briefly, *observable*) and Δ a Borel set on the real line, as in standard QM. But each observable $\mathcal{A}_0$ is obtained from an observable $\mathcal{A}$ of standard QM by adding to the spectrum Σ of $\mathcal{A}$ a *no-registration* position a_0 associated to a "ready" state of $\mathcal{A}_0$. The result a_0 is then accepted as a possible outcome in a measurement of $\mathcal{A}_0$, so that also $(\mathcal{A}_0, \{a_0\})$ is considered as a possible property of the physical object x on which the measurement is performed. Hence, obtaining a_0 is not interpreted as a failure in detecting x because of a lack of efficiency caused by the flaws of the concrete instrument, but as the registration of an intrinsic feature of x (intuitively, *x is such that it cannot move the "ready" state of $\mathcal{A}_0$ into a new state*).

(iii) For every Borel set Δ, the property $(\mathcal{A}_0, \Delta)$ is represented by the same (orthogonal) projection operator that represents $(\mathcal{A}, \Delta\backslash\{a_0\})$ in standard

QM (equivalently, ($\mathcal{A}$, Δ), since a_0 does not belong to the spectrum Σ of $\mathcal{A}$). Therefore, whenever $a_0 \in \Delta$ the properties ($\mathcal{A}_0$, Δ) and ($\mathcal{A}_0$,$\Delta \backslash \{a_0\}$) are represented by the same projection operator (similarly, if $a_0 \notin \Delta$, ($\mathcal{A}_0$, Δ) and ($\mathcal{A}_0$, $\Delta \cup \{a_0\}$) are represented by the same projection operator), though they are physically different (for instance, ($\mathcal{A}_0$, $\{a_0, a_k\}$) is the property "being not detected or having value a_k of $\mathcal{A}$," while ($\mathcal{A}_0$, $\{a_k\}$) is the property "being detected and having value a_k of $\mathcal{A}$"). Thus, not only physically equivalent, but also physically inequivalent properties are represented by the same mathematical object. In this sense, we say that *the representation of properties is not bijective in the SR model*.[4]

(iv) A binary relation of *commeasurability* is defined on the set of properties, as follows: two properties F_1 and F_2 are commeasurable iff an observable $\mathcal{A}_0$ exists, the measurement of which provides a simultaneous measurement of F_1 and F_2. Furthermore, commeasurable properties are assumed to be represented by commuting projection operators, as in standard QM.

(v) One introduces *accessible* and *nonaccessible* physical situations according to the scheme introduced in Section 3. To be precise, one says that an accessible physical situation is considered whenever a given physical object x in a state S is assumed to be detected and to possess some pairwise commeasurable physical properties; one says that a nonaccessible physical situation is considered whenever x is assumed to be not detected or to possess properties that are not pairwise commeasurable. Hence, in particular, a nonaccessible physical situation is considered whenever the outcome a_0 is assumed to occur. Moreover, *properties corresponding to the same projection operator are not physically distinguishable in an accessible physical situation.*

(vi) The probability that a given physical object in a given state possesses a given property can be evaluated by referring to the representations of states and properties and using the rules of standard QM *in all accessible physical situations*. Hence, the mathematical apparatus and the statistical predictions of QM are preserved in such situations.

(vii) For every physical object x, all properties are objective in the sense specified in Section 3, that is, they are possessed or not possessed by x independently of any measurement. Thus, for every physical situation and property $F = (\mathcal{A}_0, \Delta)$, one can associate a value $v(F) = 1$ (alternatively, $v(F) = 0$) to F if F is possessed (alternatively, not possessed) by x. Because of objectivity, properties can then be considered as hidden parameters, taking

[4] The SR model has been recently simplified, assuming that a property ($\mathcal{A}_0$, Δ) has a mathematical representation (a projection operator) only if $a_0 \notin \Delta$ (Garola and Pykacz, 2004). In this case, every projection operator corresponds to a property (in absence of superselection rules), but not all properties have a mathematical counterpart. However, the conclusions at the end of this section hold true also in the new version of the model.

values 0 or 1 (a *hidden pure state* can then be defined as an assignment of values to all properties of the system). These parameters are necessarily noncontextual, since contextuality would imply nonobjectivity.

(viii) Let $P, Q, R, \ldots$, be commuting projection operators, and let us consider an empirical physical law of standard QM expressed by a relation of the form $f(P, Q, R, \ldots) = 0$ (which is a special case of the relation $f(A, B, C, \ldots) = 0$ considered in Section 2, where $A, B, C, \ldots$ are Hermitian operators). According to the SR model, $P, Q, R, \ldots$ do not correspond bijectively to physical properties. Hence, if $F, G, H, \ldots$ are properties represented by $P, Q, R, \ldots$ respectively, one cannot generally assert that the values of $F, G, H, \ldots$ are related by $f(v(F), v(G), v(H), \ldots) = 0$ if $F, G, H, \ldots$ are not suitably chosen. But if one considers an accessible physical situation, properties represented by the same projection operator are physically indistinguishable, the choice of $F, G, H, \ldots$ is irrelevant, and one expects that the values of $F, G, H, \ldots$ satisfy the aforementioned relation. Thus, the quantum law $f(P, Q, R, \ldots) = 0$ is fulfilled in all situations in which one can actually test it, it may be violated in those situations that are not accessible to experience. It follows that the hidden parameters (properties) do not satisfy the KS condition in the SR model, but they satisfy MGP, as anticipated in Section 3.

5. SOME REMARKS ON LOCALITY

It has been already noted at the end of Section 2 that special cases of the KS-condition are understood in all existing proofs of nonlocality of QM. Thus, one may wonder whether the substitution of this condition with MGP also allows one to avoid nonlocality. It has been proven in a number of papers that the answer is positive (see, e.g., Garola and Pykacz, 2004; Garola and Solombrino, 1996b).

Furthermore, the SR model constitutes an example of HPT in which locality holds as a consequence of objectivity. It is then interesting to compare it with some different attempts to introduce local hidden variables of the kind envisaged by Bell in his original paper on the EPR paradox (Bell, 1964), but avoiding the contradiction with quantum predictions pointed out by the original Bell inequality and by all Bell-type inequalities derived later. This comparison has been briefly carried out in the paper in which the SR model was propounded, and leads one to the conclusion that the aforesaid attempts are basically different from the SR model, though there are similarities that can mislead the reader. More precisely, all local hidden variables theories implicitly require that the hidden variables satisfy constraints that are equivalent to special cases of the KS condition, so that they imply the Bell inequality, hence contradict QM. In order to avoid this contradiction a *quantum detection efficiency* can be introduced which makes it impossible to discriminate between QM and local hidden variables theories on the basis of the existing experimental results (see, e.g., Clauser and Horne, 1974; Fine, 1989; Garuccio, 2000; Szabo, 2000). But, of course, further experiments with higher efficiencies could invalidate this kind of theories if the results predicted by QM were obtained. On the contrary, the SR model introduces local hidden parameters that do not satisfy the KS condition, so that they do not imply any contradiction with QM within accessible physical situations: thus, it cannot be disproved by empirical tests. The role of the SR model is indeed purely theoretical: it aims to show that an objective (hence noncontextual and local) and physically reasonable interpretation of QM is possible, contradicting deeply-rooted beliefs and helping to avoid a number of paradoxes. Besides this, it also suggests how QM can be embodied, at least in principle, into a more general objective theory (Garola, 2003).

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